Estimation of lava tube cave heights of the Moon and the Mars from those of the Earth

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Abstract

The flow in the lava tube is modeled by Bingham fluid flowing in the inclined cylindrical pipe with gravity potential. Then, the condition of the cave formation is formulated and compared with the lava tube caves of the Earth such as those of the Mount Fuji. This formulation was applied to estimate the height of the lava tube caves of the Moon and the Mars. Gravity, lava density, slope angle and Bingham yield strength are the decisive parameters that determine the cave height.

1. Introduction

When the lava spouted from a crater flows to a foot, the flow surface and the bottom part are cooled and become solid , then, a tube structure is formed. Only the interior part of the lava flow advances then drained out from the tube. It's thought that a lava tube cave is formed in this way¹). An example of a lava tube cave is shown in Fig.1.



Photo 1 Lava tube cave of Jinza-Fuketsu No1 in Mt.Fuji

By using the simplified model of the Bingham fluid flow in the inclined pipe, the forming conditions of the lava tube cave are obtained²). The slope angle and the Bingham yield strength play the main role for the determination of the cave height³). This model has been applied to the caves of Mt.Fuji^{4,5}, other Japanese lava caves^{6,7,8,9} and lava caves of foreign countries^{2,10} on the Earth.

After introducing the study result of the lava tube cave of the Earth. this model was applied to the Moon and Mars to estimate possibility of the existence of the lava tube cave and their height. Because there is new discovery of Haruyama^{12,13,14} and later by Robinson¹⁵⁾on the Moon, the previously study¹⁶⁾was revised.

2. Lava tube cave formation conditions and previous studies on the Earth

Concept of Lava tube and cave formation is shown in Fig.1. (A)The lava supplied from the underground will get over the edge of the crater, and flow down though the slope to the foot. (B) The cooled surface of lava flow becomes solid and inner fluid lava will drain out when the supply of the lava from the crater is terminated. (C) Then, lava tube cave like a tunnel will be formed.

In modeling the discharge mechanism of this type of lava tube, we used an inclined circular tube model for the sloping section of the cave as shown in Fig.2. A simple model of steady state isothermal laminar flow in inclined circular pipes was used for analyses²).

Flow characteristics were studied as a function of parameters such as tube radius, viscosity, yield strength of lava and slope inclination. Here, f_B is Bingham yield stress, η_B is Bingham viscosity, which takes specific value depending on the materials.

For laminar flow model in circular tube on the slope, the equation of the distribution of flow speed u of Bingham fluid are shown as follows^{2,3}

For
$$\tau_{\rm w} = (\rho g \sin \alpha) R/2 > f_{\rm B}$$

$$u=(R-r_B)^2(\rho g \sin \alpha)/4\eta_B$$
 $r < r_B$

 $u=[R^2-r^2-2r_B(R-r)](\rho g \sin \alpha)/4\eta_B \qquad r>r_B$

for $\tau_w = (\rho g \sin \alpha) R/2 < f_B \ c t$

Here, α is angle of slope or inclination of tube, ρ : density of the fluid, g: gravity acceleration, R: radius of the tube, r_B: radius of the flowing position where Bingham yield stress takes f_B.



Fig.1 Concept of Lava tube and cave formation



Fig.2 Flow speed distribution in the inclined pipe:Plug flow for Bingham fluid.

 $(\rho g \sin \alpha)R/2=f_B$ is the limiting condition to see for the lava to be drained out or to be plugged in the tube.

For the given ρ , slope angle and f_B , the cave height H=2R,H=2R=4 f_B / ($\rho g \sin \alpha$) is given³). On the contrary, For the given, ρ , slope angle and the cave height H=2R, the yield strength f_B can be obtained³).

As the typical examples for various lava cave topography $^{11)}$ of the caves of Mt. Fuji 4,5), other japanese caves^{6,7,8,9)} and those of foreign countries^{2,10)}, the

plotted data for the slope angle and the cave height are indicated in Table1~6 and Fig 3~8. As seberal yield strengths of the lava are also indicated, the yield strength area of the lava of each cave can be seen. These obtained yield strengths are reasonable value in comparison with those of G.Hulme¹⁷⁾ as shown in Table 7.

Fig.9 shows a relation between obtained yield strength and SiO_2 wt% for the each lava. Summary of the results are shown in Table 9.

Table1 Slope angle and cave height of lava tube caves of Mt.Fuji

(*extracted from T.Honda(2001):Formation Mechanism of Lava Tube cave in Mt.Fuji,Fall meeting of Volocanological Society of Japan,p66)

Cave name	Slope angle	Inner height
Subashiri-tainai Upper part	20°	1m
Subashiri-tainai Lower part	15°	2m
Jinza-Fuketsu No1	13°	5-10m
Jinza-Fuketsu No3	11.5°	5m
Shoiko-Fuketsu No1A	10°	3.3m
Shoiko-Fuketsu No1B	7.6°	2m
Karumizu-Fuketsu	5.5°	4m
Fuji-Fuketsu No1	8.1°	10m
Motosu-Fuketsu No1	3.6°	10m
Inusuzumiyama-Fuketsu No1	12°	5m
Mujina -Ana	8.5°	5m
Inusuzumiyama-Fuketsu No2	13°	2m
Mitsuike-Ana	3.2°	10m
Atsuhara-Fuketsu	10°	2m
Banba-Ana	4.8°	5-10m



Fig.3 Slope angle and cave height for Mt.Fuji

Table2 Slope angle and lava tube cave height of Mt.Etna (*extracted from Sonia Calvari,MarcoLiuzzo(1999): Excursion guide,Lava tubes and Lava cavern Etna volcano,9th Int.Symp.Vulcanospeloeology)

Cave name	Length	Denive	Slope	Inner
		lation	angle	height
Cutrona Cave	870m	97m	6.4°	6m
Immoacolatella I	300m	20m	3.8°	10m
Cave				
Serracozzo Cave	350m	60m	9.8°	2-3m
Tre Livelli Cave	1150m	304m	15.3°	3m
KTM Cave	643m	100m	8.9°	5m
Cassone Cave	246m	1 m	0.23°	7m
Intraleo Cave	300m	16m	3.1°	9-3.8m
Abisso di Monte	756m	84m	6.4°	10m
Nero Cave				



Fig.4 Slope angle and cave height for Mt.Etna

Table 3 Slope andgle and cave height for Kazumura cave of Kirauea(*extracted from K.Allred,C.Allred(1997):

Development and Morphology of Kazumura Cave, J. Cave& Karst Studies, 59(2), pp67-80)

Portion of cave	Average slope angle	Estimated erosion
		depths
Olaa	2.5°	5.6-19.9m
Sexton	2.0°	3.4-17.2m
Upper	1.7°	4.0-11.1m
Old	1.9°	3.4-10.1m
Lower	1.3°	3.4-10.1m



Fig.5 Slope angle and cave height for Kazumura cave of Kilauea

Table4 Slope angle and cave heigh of Mt.St.Helens (*extracted fromJ.H.Hyde &R.Greeley(1973):Geological

Field Trip Guide,Mount St.Helens lava tubes,Washington)					
Cave Name	Slope Angle	Maximum Height			
Ape Cave	3.3°	11.6m			
Barney's Cave	2.0°	2.7m			
Bat Cave	16.2°	3.7m			
Beaver cave	3.0°	9.1m			
Flow Cave	3.2°	2.4m			
Lake Cave	2.6°	15.5m			
Little People Cave	4.5°	9.1m			
Ole's Cave	2.1°	7.6m			



Fig.6 Slope angle and cave height for Mt.St.Helens

Table 5 Slope angle and cave height of Suchiooc, Mexico (*extracted from Ramon Espinasa Perena(1999),

Thesis,Origen y evolucion de tubos de lava en la Sierra

Chichinatzin:El caso del volcan Suchiooc,Univ.Nacional Autonoma de Mexico)

ue mexico)				
Cave name	Length	Denive	Slope	Inner
		lation	Angle	Height
Cueva de Aucomolijia	343m	44m	12.9°	4m
Cueva de Barreto	129m	24m	10.7°	6m
Cueva de Tepemecac	278m	38m	7.9°	9m
Cueva de la Tuberia	428m	116m	15.8°	5m
Cueva de Tiro Perdido	262m	22m	4.8°	15m
Cueva del Arbol	1480m	118m	4.6°	15m
Sistema Chimalacatepec	1388m	201m	8.3°	10-15m



Fig.7 Slope angle and cave height for Schiooc in Mexico





Table 7	Comparison	between yield strength obtained
by ca	ve height and t	hat obtained by other method

by cave neight and that obtained by other method			
Volcano	SiO2 wt%	Yield strength obtained from cave	
		height (Yield strength obtained	
		from other method)	
Mt.Fuji	49~51%	$1x10^{4}$ ~7.5x10 ⁴ dyne/cm ²	
Mihara-yama,	52~53%	$5.0 \times 10^4 \text{dyne/cm}^2$	
Izu-Oshima		$(4.3 \times 10^4 \text{dyne/cm}^2 \text{ by Hulme}^{17})$	
Mt.Etna	48%	$1 \times 10^{4} \sim 5 \times 10^{4} \text{dyne/cm}^{2}$	
		$(7 \times 10^4 \text{dyne/cm}^2 \text{ by Hulme}^{17})$	
Kilauea	47~50%	$2.5 \times 10^{3} \sim 2.5 \times 10^{4} \text{dyne/cm}^{2}$	
		$(1 \times 10^3 \text{dyne/cm}^2 \text{ by Hulme}^{17})$	
Piton de la	48%	$5x10^{3}$ ~7.5x10 ⁴ dyne/cm ²	
Fournaise			
Mt.St.Helens	50%	$5x10^{3} \sim 2.5x10^{4} dyne/cm^{2}$	
Cameroon	43.5%	$7.5 \times 10^4 \sim 1.0 \times 10^5 dyne/cm^2$	
		(~1x10 ⁵ dyne/cm ² by Fitton ²³⁾)	
Suchiooc	51%	$2.5 \times 10^4 \sim 1.0 \times 10^5 dyne/cm^2$	



Fig.9 Relation between yield strenght and SiO₂wt%

3. Estimation of lava tube cave heights of the Moon and the Mars

If the yield strength and the angle of the slope are known, the height of the lava tube cave can be estimated by applying this model to the Moon and Mars. As shown in Table 8, The main difference with the Earth is only gravity. The lava density is same as 2.5 g/cm³. The yield strength is indicated from the thickness of the lava and the surface levee estimated by the Hulme^{17,18,19}, Moore^{20,21)} and Zimbelman²²⁾. The details are shown in Table 10.

Table 8 Physical conditions

		5	
Planet	Gravity	Lava density	Yield strength
Earth	9.8 m/s^2	2.5 g/cm^{3}	2x10 ³ dyne/cm ²
			$\sim 1 \mathrm{x} 10^5 \mathrm{ dyne/cm^2}$
Moon	1.62 m/s^2	2.5 g/cm^{3}	1×10^3 dyne/cm ²
			$\sim 2 \mathrm{x} 10^5 \mathrm{ dyne/cm}^2$
Mars	3.71 m/s^2	2.5 g/cm^{3}	$3x10^3$ dyne/cm ²
			\sim 3x10 ⁵ dyne/cm ²

Fig.10 and Fig.11 show the relation of slope angle and cave height $H=2R=4f_{B}/(\rho g \sin \alpha)$ for different yield strength for the Moon and the Mars.



Fig.10 Lava tube cave height estimation for the Moon



Fig.11 Lava tube cave height estimation for the Mars

Regarding the Moon, according to Hulme¹⁷⁾, Mare Imbrium has 0.2 deg of slope angle and it's yield strength is $4x10^3$ dyne/cm². The possible cave height can be estimated as 112m from Fig.10. As the lava thickness is about 30m for Mare Imbrium, the formation of lava tube cave would be difficult in this area. On the other hand, Marius Hills where an opening was found by Haruyama^{12,13)}, has an altitude of 1-2km and a diameter are 300km, so the degree of slope angle is approximately 0.38 deg~0.76 deg, so if the same yield strength of $4x10^3$ dyne/cm² is used, the cave height will be 30~60m from Fig.10. So, if the thickness of the lava is more than 30~60m, there is a possibility that a lava tube cave is formed in this area.

Regarding the Mars, according to Zimbelman ²²⁾, Acraeus Mons has 5 deg of average slope angle and yield strength of 2.1×10^4 dyne/cm². The lava tube cave height can be estimated as about 10m from Fig.11. As the lava thickness is said to be 15-45m, so there is a possibility that a lava tube cave is formed. Pavonis Mons has an altitude of 8.7km and the diameter is 375km, so the degree of slope angle is 2.66 deg. If the same value of yield strength of 2.1×10^4 dyne/cm² is used, the lava tube cave height can be estimated as 19m from Fig.11. The formation lava tube cave would be possible if the lava thickness is higher than 19m at Pavonis Mons.

If the more precise degree of slope angle, lava thickness and yield strength of the lava flow are known, it is possible to predict more precisely the cave height.

4. Conclusions

If the lava tube caves of the earth and of other planet such as the Moon and the Mars are formed from the same mechanism, the lava tube cave height of the Moon and the Mars can be predicted by the same model. The main difference remains in the gravity. The lava density and the lava yield strength seem not be so different. Other elements are slope angle and lava thickness. In Table 9 and 10, Summarized values are shown for the Earth, the Moon and the Mars.

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 Table 6 Lava tube caves in the Reunion Island

 (*from ref.1 and ref.2 for length and denivelation)

Ref-1: P.Audra(1997) : Inventaire preliminare des cavernes de l'ile de la Reunion. Spelunca, no.66, p23-38, Edition FFS.

Ref-2: D.Cailhol,S.Fulcrand(2011) :Mission d'Expertise: Tunnel de laves de l'ile de la Reunion

Cave name	Length	Denive	Location/area	Slope	Inner
		lation		angle	height
∘Tunnel de 2004:Branche nord	741m	44m	East, Piton de la Fournaise	3.4°	2m
∘Tunnel de Dimanche	275m	24m	East, Piton de la Fournaise	5.0°	1-2m
○Tunnel de Gendarmes	340m	25m	East, Piton de la Fournaise	4.2°	2-4m
∘Tunnel de Brule de Citron-galets	680m	94m	East.Piton de la Fournaise,Saint-Philippe	6.1°	2-4m
∆Caverne Bateau	1910m	34m	Central,Le Tampon	1.0°	2-10m
∆Caverne de Butor	100m	4m	Central,Saint-Joseph	2.3°	2m
∆Caverne des Fees	820m	26m	Central,(Piton des Fees),La Plaine des Palmistes	1.8°	2-4m
∆Trou no2 de la Plaine-des-Palmistes	947m	30m	Central,La Plaine des Palmistes	1.8°	2-4m
∆Caverne de Pylone,Amont	80m	2m	Central, La Tampon	1.4°	2m
∆Caverne de Pylone,Aval	160m	6m	Central, Le Tampon	2.2°	2m
©Caverne de la Ravine Saint-Francois	165m	47m	Central plateau, (Piton des Cabris), La Plaine des	14.7°	2-5m
			Palmistes		
© Trou du Sentier de Piton Textor	175m	48m	Central Plateau, (Piton Textor) Le Tampon	15.9°	4-6m
▲ Le Trou d`eau	350m	45m	West, Piton des Neiges Saint-Paul	7.4°	5m
▲ Caverne de Bernica	369m	31m	West, Piton des Neiges, Saint-Paul	4.8°	2-4m
▲ Caverne de la Ravine Fleurimont	200m	9m	West, Piton des Neiges Saint-Paul	2.6°	2-6m
▲ Caverne de Quatre Voies	136m	11m	West, Piton des Neiges, Saint Paul	4.6°	4-5m
▲ Grotte des Salanganes	550m	21m	West, Piton des Neiges Saint-Paul	2.1°	0.5-5m

	Table 9 . Eava table caves of the Earth				
Planet	Location of lava flow/lava tube	Slope angle of	Yield strength obtained from	Height of	
	cave(Reference)	lava tube cave	lava tube cave height	lava tube	
Earth	Mt.Fuji(T.Honda ⁴⁾)	3.2 °~20.0 °	$1 \times 10^{4} \sim 7.5 \times 10^{4} \text{dyne/cm}^{2}$	1~10m	
	Mihara-yam.Izu-Oshima(T.Honda ^{8,9)})	~30 °	$5.0 \times 10^4 \text{dyne/cm}^2$	~1.5m	
	Hachijou-jima,Nishi-yama(T.Honda ^{6,7)})	4.0 °~14.0 °	2.5×10^4 dyne/cm ²	2~5m	
	Kilauea(T.Honda ³⁾)	1.0 °~4.0 °	$2.5 \times 10^3 \sim 2.5 \times 10^4 \text{dyne/cm}^2$	3~17m	
	Mt.Etna(T.Honda ³⁾)	0.2 °~15.3 °	$1 \times 10^4 \sim 5 \times 10^4 dyne/cm^2$	2~10m	
	Mt.St.Helens(T.Honda ³⁾)	2.1 °~4.5 °	$5x10^{3} \sim 2.5x10^{4}$ dyne/cm ²	3~16m	
	Piton de la Fournaise(T.Honda etal ¹⁰)	1.0 °~16.0 °	$5x10^{3}$ ~7.5x10 ⁴ dyne/cm ²	1~12m	
	Suchiooc(T.Honda ³⁾)	4.6 °~15.8 °	$2.5 \times 10^4 \sim 1.0 \times 10^5 \text{dyne/cm}^2$	4~15m	
	Cameroon(T.Honda ³⁾)	~14.0 °	$7.5 \times 10^4 \sim 1.0 \times 10^5 \text{ dyne/cm}^2$	6~8m	

Table 9 : Lava tube caves of the Earth

Table 10: Lava tube caves of the Moon and the Mars

Planet	Location of lava flow(Reference)	Slope angle of lava flow	Yield strength obtained by lava	Estimated
			flow configuration	cave height
Moon	Mare Imbrium(Hulme ¹⁷⁾)	0.2 °	$4x10^3$ dyne/cm ²	(112m)
	Imbrium flow(Moore/Schaber ²⁰⁾)	0.13°,sinα=0.0023 0.38°~	$1 \sim 2 \times 10^3$ dyne/cm ²	
	Marius Hills(elevation	0.76 °	$(4x10^3 \text{ dyne/cm}^2)$	(30~60m)
	/diameter:1-2km/300km)			
	King Crater(Moore ²¹⁾)	sina=0.08~0.22	(2.41 ± 1.71) x10 ⁵ dyne/cm ²	
	Aristarchus Crater(Moore ²¹⁾)	sina=0.12~0.36	(1.94 ± 1.13) x10 ⁵ dyne/cm ²	
	Aristarchus Crater(Hulme ¹⁹⁾)	sina=0.15~0.66	$1.0 \times 10^4 \sim 1.3 \times 10^5 \text{dyne/cm}^2$	
	Copernicus(Hulme ¹⁹⁾)	sina=0.09	1.8×10^5 dyne/cm ²	
	Tycho(Hulme ¹⁹⁾)	sina=0.04~0.10	$4.0x10^{3} \sim 1.0x10^{5} dyne/cm^{2}$	
Mars	Ascraeus Mons(Zimbelman ²²⁾)	3.5 °~6 °	$3.3 \times 10^3 \sim 8.3 \times 10^4 \text{dyne/cm}^2$	
		Average:5 °	$Av:2.1x10^4 dyne/cm^2$	(10m)
	Olympus Mons(Hulme ¹⁸⁾)	sinα=0.04	(8.8 ± 1.3) x10 ³ dyne/cm ²	
		sina=0.06	(2.0 ± 0.3) x10 ⁴ dyne/cm ²	
		sina=0.09	(4.5 ± 0.6) x10 ³ dyne/cm ²	
	Olympus Mons(Moore ²¹⁾)	sina=0.082,0.089	Av:(3.06±1.24)	
			$x10^{5}$ dyne/cm ²	
	Arsia Mons(Moore ²¹⁾)	sina=0.021~0.070	Av:(1.00±0.82)	
			$x10^4$ dyne/cm ²	
	Pavonis Mons(elevation	2.66 °	$(2.1 \times 10^4 \text{dyne/cm}^2)$	(19m)
	/diameter:8.7km/375km)			